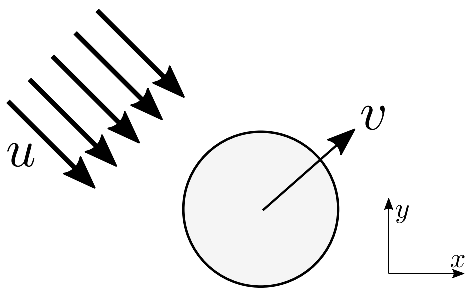
Chapter 2: Flow of a single particle

Exercise 1

A spherical particle (diameter 100 μm) moves in water with a density of 1000 kg/m^3 and dynamic viscosity of 0.001 Pa·s. The particle velocity can be described by the vector: $\mathbf{v} = 2.0\mathbf{i} + 2.0\mathbf{j}$ [m/s], while the fluid velocity is: $\mathbf{u} = 3.0\mathbf{i} - 3.0\mathbf{j}$ [m/s].



Find the drag force acting on the particle, and find the force direction.

Solution:
$\mathbf{F}_D = 1.12 \cdot 10^{-5}\mathbf{i} - 5.6 \cdot 10^{-5}\mathbf{j}$ [N]

Exercise 2

A spherical and stationary particle was put into moving water (with a density of 1000 kg/m^3 and dynamic viscosity of 0.001 Pa·s). The water speed is 1.0 m/s.

The particle diameter is 0.5 mm, and its density is 2000 kg/m^3.

As a result, the particle starts to accelerate. Write particle's equation of motion assuming that C_D is constant and equal to 0.44 (Newton's law).

Solve this equation and find the particle velocity after 0.05 s.

Solution:
0.9375 m/s

Exercise 3

In this exercise, we analyse the Basset force. A flat plate undergoes acceleration described by the relation: $v = at^2$ (i.e., a "parabolic" acceleration). The coefficient a is known.

The fluid above the plate has material properties characterized by ρ_f and μ_f.

Find the shear stress between the plate and the fluid as a function of time.

Solution:
$\tau_w(t) = \frac{8}{3}a\sqrt{\frac{\rho_f \mu_f}{\pi}}t^{3/2}$

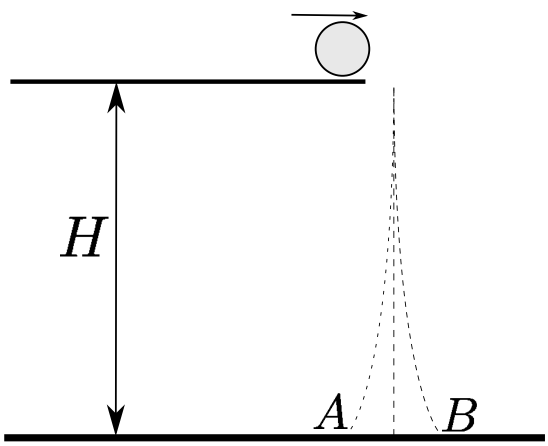
Exercise 4

Repeat the previous exercise by assuming that the plate accelerates according to the following relation: $v = at^3$.

Solution:
$\tau_w(t) = \frac{16}{5}a\sqrt{\frac{\rho_f \mu_f}{\pi}}t^{5/2}$

Exercise 5

A spherical ball with a radius of 1 cm is pushed off a table and begins rotating clockwise at an angular velocity of 3 rad/s as it falls.



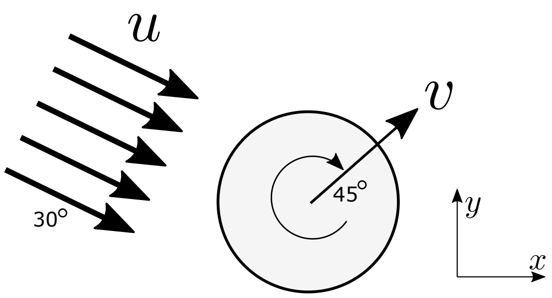
(a) Where is the ball going to land, in point A or point B (due to the Magnus force)?

(b) Calculate the distance between the landing point and vertical line. Assume that there is no air resistance, but consider the Magnus force. Height H is equal to 1.0 m. The ball material density is 1500 kg/m^3, and air density is 1.2 kg/m^3.

Solution:
(a) point A; (b) 0.27 mm

Exercise 6

A spherical particle (diameter 2 mm) moves with a velocity, whose magnitude is $v = 2$ m/s. The particle also rotates clockwise with an angular velocity of 0.5 rad/s, as the figure below shows.



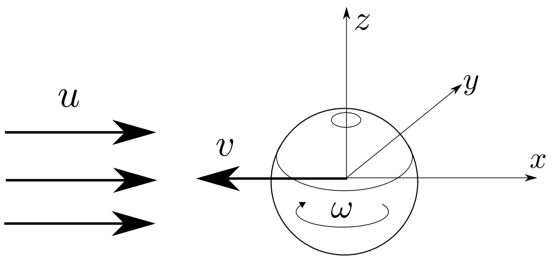
The particle is subjected to a fluid flow with the density of 1000 kg/m^3 and dynamic viscosity of 0.001 Pa·s (water). The velocity magnitude is $u = 4$ m/s.

Calculate the components of the drag and Magnus force acting on the particle.

Solution:
$\mathbf{F}_D = 5.642 \cdot 10^{-3}\mathbf{i} - 9.395 \cdot 10^{-3}\mathbf{j}$ [N]
$\mathbf{F}_M = 5.363{\cdot}10^{-6}\mathbf{i} + 3.22{\cdot}10^{-6}\mathbf{j}$ [N]

Exercise 7

A spherical particle with diameter of 4 mm is placed in a flow as shown in the figure.



The particle moves with a speed $v = 0.1$ m/s in the negative direction of x-axis. At the same time, the fluid flows in the positive direction with a speed $u = 0.2$ m/s. The fluid being used is water (density is 1000 kg/m^3 and dynamic viscosity is 0.001 Pa·s).

In addition, the particle rotates around the z-axis with an angular velocity of $\omega = 0.2$ rad/s (see the figure).

Calculate the components of the drag and Magnus force acting on the particle.

Solution:
$\mathbf{F}_D = 4.976 \cdot 10^{-4}\mathbf{i}$ [N]
$\mathbf{F}_M = 1.508 \cdot 10^{-6}\mathbf{j}$ [N]